MathLab. (updated) 10/11

Introduction to Autonomous Mobile Robots - SIEGWart.
Nourbakhsh
Scaramuzza.

Matlab or octave
Simulations V-Rep.

Matrix equation:

$AX = B$

Put constants on one side.

$\Downarrow$

$$\begin{aligned} 1.5x_1 + x_2 &= 3 \\ x_3 &= 4x_2 \\ 4 - x_1 + x_2 &= x_3 \end{aligned} \Rightarrow \begin{aligned} 1.5x_1 + x_2 \quad &= 3 \\ -4x_2 + x_3 &= 0 \\ -x_1 + x_2 - x_3 &= -4 \end{aligned}$$

col. of unknowns

$$\underset{A}{\begin{bmatrix} 1.5 & 1 & 0 \\ 0 & -4 & 1 \\ -1 & 1 & -1 \end{bmatrix}} \underset{X}{\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}} = \underset{B}{\begin{bmatrix} 3 \\ 0 \\ -4 \end{bmatrix}}$$

$AX = L$

So.

$X = A \setminus B$

$X = \frac{A}{B}$

Key

$$\begin{aligned} AX &= B \\ A^{-1} \cdot AX &= A^{-1} \cdot B \\ IX &= A^{-1} \cdot B \\ X &= A^{-1} \cdot B \end{aligned}$$

MatLab commands

```
>> X = A\B
   X = 1.4286
       0.5571
       5.4286
```

$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$

MatLab Variables

▦ A <3x3 double>
▦ B <3x1 double>
▦ X <3x1 double>

Larger Bridge data (solved almost same)

Matlab Commands

```
>>load bridgeData
>>tension = Coeff\Force
tension = 1.2516
          0.9787
          2.0737
          0.8501
          1.4542
```

Matlab Variables

```
⊞ Coeff    <300 x 300  double>
⊞ Force    <300 x 1    double>
⊞ tension  <300 x 1    double>
```

MathLab Calculator 10/11

```
>> a = 3;
>> b = 2;
>> c = -6;
>> x1 = (-b + sqrt(b^2 - 4xaxc)) / (2xa)
```

Functions:

1 column

[a, I] = min(x) is ⊞ $\begin{bmatrix} 0 \\ 1 \\ 0 \\ -1 \\ 0 \end{bmatrix}$ 4

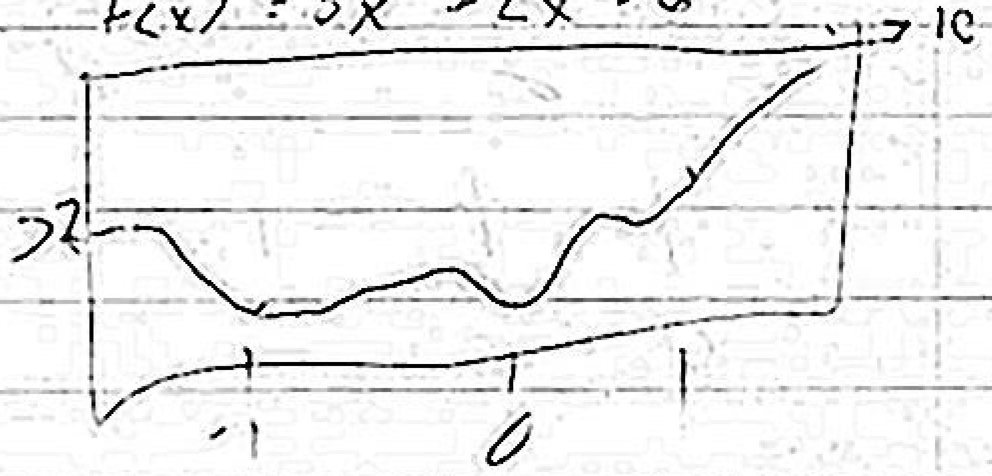
⊞ a = -1 ⊞ I = 4

Graph

plot(x, y)

```
>> x = [-2, -1, 0, 1, 2];
>> y = [2, -5, -6, -1, 10];
>> plot(x, y)
```

F(x) = $3x^2 + 2x - 6$



Lookup MathLab Functions
Documentation > Support

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Using Online Documentation
www.mathworks.com/help.
Documentation > SearchResults
ex. Standard Deviation Calc.

MathLab comm.

```
% Find stand. dev of autosales.
avgSales = mean(salesData);
stnDev = std(salesData);
```

MathLab Var.

⊞ salesData <6x6 double>
⊞ avgSales <1x6 double>
⊞ standDev <1x6 double>

creating uniformly spaced vectors.

x = startValue : spacing : endValue;

```
>> x = -2 : 0.15 : 2;
```

X

⊞ x [-2, -1.85, -1.70, ... 1.75, 1.90]   2.05

matrix

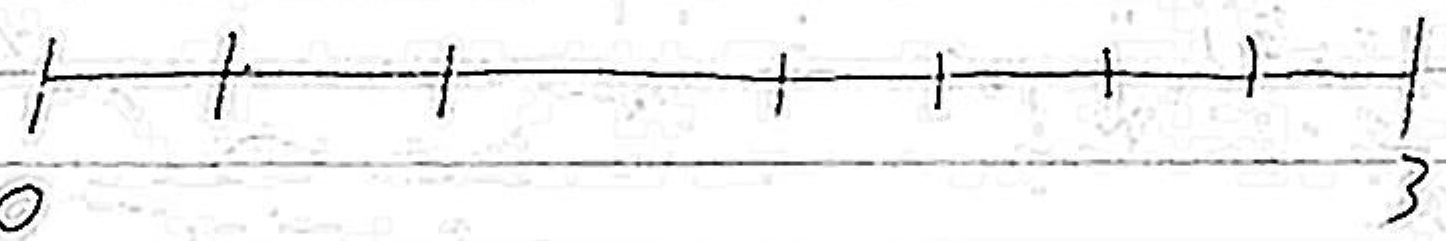
⊞ x $\begin{bmatrix} -2 \\ -1 \\ 0 \\ 1 \\ 2 \end{bmatrix}$

start end

```
>> x = (-2 : 2)' ;
```

5-by-1 column vector

creating uniformly spaced vectors LINSPACE



8 points

⊞ v = [0 0.43 0.86 1.23 1.71 2.14 2.6 3]

```
>> v = linspace(0,3,8)
```

creating Matrices:

use commas to create row vectors

use semicolons to create column vectors

```
>> c = [1;2;3];
```

; indicates new row

⊞ c = $\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$

```
>> M = [1,2,3; 4,5,6]
```

$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}$: M ⊞

matlab: (vector & matrix operations) 10/12

Determining Array Size & Length.

length - use on Vectors
size - use on matrices

MATLAB Comm,	Matlab Variables
>> n = length(v)	⊞ v <1x24 double>
n = 24	⊞ A <24x30 double>
>> [nrows, ncols] = size(A);	⊞ nrows 24
& matrix	⊞ ncols 30

Ex. Matlab comm

```
p = load('pollutionData.mat');
[nrows, ncols] = size(P);
% Fit a trendline to each column
xData = 1 : nrows;
for i = 1 : ncols
    yData = p(1 : nrows, i);
```

Accessing Elements of a Vector

Matlab Comm. (reference last value in vector)

```
>> early = 1:3
           1-3 as columns
>> afternoon = data(early);
>> evening = data(end-2 : end);
                  ↑
                lastVal
```

figure (6-2) 4

data = [18, 25, 43, 49, 55, 48]

(end-2)(end-1)(end)

afternoon = [18, 25, 43]

Calculations with Vectors

MATLAB = MATRIX LABoratory

*
^ } perform matrix operations
/

Ex. v1 [1, 2, 3, 4]
v2 [2, 4, 6, 8]

v1 * v2 = [1*2, 2*4, 3*6, 4*8]
v1 / v2 = [1/2, 2/4, 3/6, 4/8]
v1 ^ 2 = [1^2, 2^2, 3^2, 4^2]

Ex. $f(x) = 3x^2 + 2x - 6$

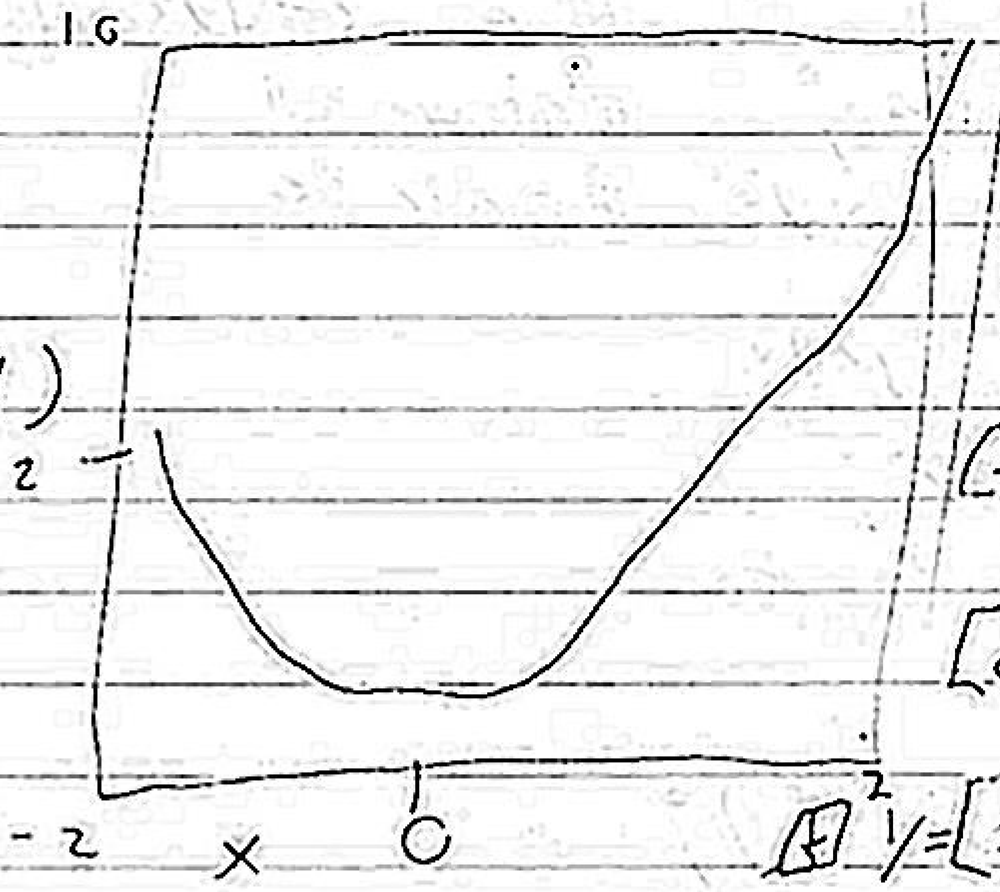
MATLAB Comm.

```
>> x = -2 : 0.1 : 2 ;
```

start   increase by   end

```
>> y = 3*x.^2 + 2*x - 6 ;
```

```
>> Plot(x,y)
```



Equates to:

$$3 \times [-2, -1.9, \ldots, 1.9, 2]^2$$

$$+$$

$$2 \times [-2, -1.9, \ldots, 1.9, 2]$$

$$-$$

$$6$$

Following order of Opperations

$$[12, 10.83, \ldots, 10.83, 12]$$
$$+ \quad + \quad + \quad + \quad +$$
$$[-4, 3.8, \ldots, 3.8, 4]$$
$$- \quad - \quad - \quad - \quad -$$
$$[6, 6, \ldots, 6, 6]$$

⊞ $y = [2, 1.63, \ldots, 8.83, 10]$

Vector Transpose (convert Row Vector to Column Vector)

⊞ $v = [2 \ -1 \ 8.5 \ 6 \ 19]$

⊞ $c = \begin{bmatrix} 2 \\ -1 \\ 8.5 \\ 6 \\ 19 \end{bmatrix}$

MATLAB Commands

```
>> c = v';
```

MathLab 10/12

Matrix Creating Functions:

· eye Identity matrix	· One Matrix of all 1's
· zeros matrix of all zeros	· rand uniformly distributed random numbers
· randn Normally distributed rand nums.	· randi uniformly distributed random integers.
· diag Diagonal matrix	· linspace Evenly spaced Vector

ex. matlab Comm.

```
>> I = eye(10);
```

$$I \boxed{+} = 10 \begin{bmatrix} 1 & 0 & 0 & & & \\ 0 & 1 & 0 & & 0 & \\ 0 & 0 & 1 & 0 & 0 & \\ & & & 1 & & \\ & & & & & 1 \end{bmatrix}$$

$$10$$

For columns

```
        rows  cols
          ↓    ↓
>> Z = zeros(10, 3)
```

$$\boxed{+} Z = 10 \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ \vdots & & \\ 0 & 0 & 0 \end{bmatrix}$$

$$\uparrow$$

$$3$$

MATLAB (Vector & matrix Operations) 10/13

Accessing Elements of a Matrix

MATLAB Comm.

```
>>rows = [1 5];
>>cols = 2:4;
>>subdata = data(rows, cols);
```

$$\begin{array}{c|ccccc} & 1 & 2 & 3 & 4 & 5 \\ \hline \textcircled{1} & 18 & 32 & 26 & 28 & 46 \\ 2 & 25 & 42 & 35 & 30 & 52 \\ 3 & 43 & 44 & 37 & 56 & 54 \\ 4 & 49 & 38 & 57 & 54 & 55 \\ \textcircled{5} & 55 & 48 & 61 & 69 & 62 \\ 6 & 48 & 34 & 56 & 42 & 56 \end{array}$$

= ⊞ data

$$\begin{bmatrix} 32 & 26 & 28 \\ 48 & 61 & 69 \end{bmatrix}$$

= ⊞ subdata

Shortcut

```
>>col1 = data(1:6, 1);
```

same As

```
>>col1 = data( :,1 );
```

$$\begin{array}{c|c} 1 & 18 \\ 2 & 25 \\ 3 & 43 \\ 4 & 49 \\ 5 & 55 \\ 6 & 48 \end{array}$$

= ⊞ col1

Accessing Elements of a Vector Using Conditions.

e.x.

⊞ data		☑ ind
0.01	>0.1	0
0.02	>0.1	0
0.01	>0.1	0
0.27	>0.1	1
0.39	>0.1	1
0.01	>0.1	0

MATLAB comm.

```
>> ind = data >0.1;
>> activity = data(ind);
```

$$\begin{bmatrix} 0.27 \\ 0.31 \end{bmatrix}$$

⊞ activity

```
% Now change all values <=0.1%
>>ind != data <=0.1;
>> data(ind) = 0;
```

- < , >
- ~ (Not)
- == , ~=
- &
- |

```
>>peak = data(data >0.4);
>>active = data(data <= 0.4 & data >0);
```

Calculations with Matrices

Ex. MATLAB Variables

⊞ quantity <3x2 double>
⊞ Price <3x2 double>
⊞ shipCost <3x2 double>
⊞ taxRate <1x1 double>

$$\begin{array}{l|cc} & \text{Supplier 1} & \text{Supplier 2} \\ \hline \text{Product 1} & 1.25 & 2.10 \\ \text{Product 2} & 4.60 & 4.15 \\ \text{Product 3} & 11.95 & 10.80 \end{array}$$

Total Cost = (Price * Quantity) * TaxRate + Shipping Cost

$$\overset{\text{Price}}{\begin{bmatrix} 1.26 & 2.10 \\ 4.60 & 4.15 \\ 11.95 & 10.80 \end{bmatrix}} \underset{\text{ele-wise } .*}{*} \overset{\text{Quantity}}{\begin{bmatrix} 5 & 5 \\ 3 & 3 \\ 2 & 2 \end{bmatrix}}$$

$$\begin{bmatrix} \overset{P}{1.25} \times \overset{Q}{5} & \overset{P}{2.10} \times \overset{Q}{5} \\ 4.60 \times 3 & 4.15 \times 3 \\ 11.75 \times 2 & 10.80 \times 2 \end{bmatrix}$$

$$\overset{\text{scalar}}{*}$$

1.05 ⊞ taxRate

$$\begin{array}{cc} P*Q & P*Q \\ \hline 6.25 & 10.50 \\ 13.50 & 12.45 \\ 23.90 & 21.60 \end{array}$$

⊞ total

Element-Wise Operations
- matrices must be of same size
- Addition: +
- Subtraction: -
- multiplication: .*
- Division: ./
- Exponentiation: .^

Applying scaler opperations to matrices
- Don't: create a new matrix & use ele-wise opps.
- Do: Use scaler Opperators (+ - * / ^)

$$\begin{bmatrix} 6.25 & 10.50 \\ 13.80 & 12.45 \\ 23.10 & 21.60 \end{bmatrix} * 1.05 \iff \begin{bmatrix} 6.25 * 1.05 & 10.50 * 1.05 \\ 13.80 * 1.05 & 12.45 * 1.05 \\ 23.90 * 1.05 & 21.60 * 1.05 \end{bmatrix}$$

```
>> total = price .* quantity;
>> total = total * taxRate;
>> total = total + shipCost;
```

$$\begin{array}{l|cc} & \text{Supp. 1} & \text{Supp. 2} \\ \hline \text{Product 1} & 14.56 & 14.02 \\ \text{Product 2} & 20.49 & 16.06 \\ \text{Product 3} & 31.10 & 28.67 \end{array}$$

MATLAB (vector & Matrix opperations) 10/13

Matrix Multiplication

Rules ④

$$\begin{bmatrix} 4 & 2 & 0 & -1 \\ 1 & 1 & -4 & 0 \\ -1 & 0 & 3 & 6 \end{bmatrix} \times \begin{bmatrix} -1 & 0 \\ 2 & 3 \\ -2 & 1 \\ 0 & -1 \end{bmatrix} = \begin{bmatrix} \boxed{0} & \boxed{2} \\ \boxed{} & \boxed{-1} \\ \boxed{} & \boxed{} \end{bmatrix}$$

3-by-④ ④-by-2 ④ 3-by-2

Row so cols

#cols = #Rows

$4 \times -1 \quad -4$
$2 \times 2 \Rightarrow 4$
$0 \times -2 \quad 0$
$-1 \times 0 \quad + 0$
$\overline{\quad 0 \quad}$

$3 \times 0 \quad 0$
$1 \times 3 \Rightarrow 3$
$-4 \times 1 \quad -4$
$0 \times -1 \quad + 0$
$\boxed{-1}$

$4 \times 0 \quad 0$
$2 \times 3 \Rightarrow 6$
$0 \times 1 \quad 0$
$-1 \times -1 \quad + 1$
$\boxed{7}$

Next Row

MATLAB Commands

```
>> A = [5,2,3;1,1,0;2,1,4];
>> x =  [-1;2;1];
>> b= A x X;
```

$$\begin{array}{ccc} A & ① & X \\ \begin{bmatrix} 5 & 2 & 1 \\ 1 & 1 & 0 \\ 2 & 1 & 4 \end{bmatrix} & \times & \begin{bmatrix} -1 \\ 2 \\ 1 \end{bmatrix} ③ \end{array}$$

cols 3 = Rows 3

If

X × 3
3-by-1 ≠ 3-by-3
Error dimension must agree.

number of Rows must equal
number of columns
⇓
Error >> X × A
⇑ ⇑
1 ≠ 3
Row Cols.

Ex. Fig

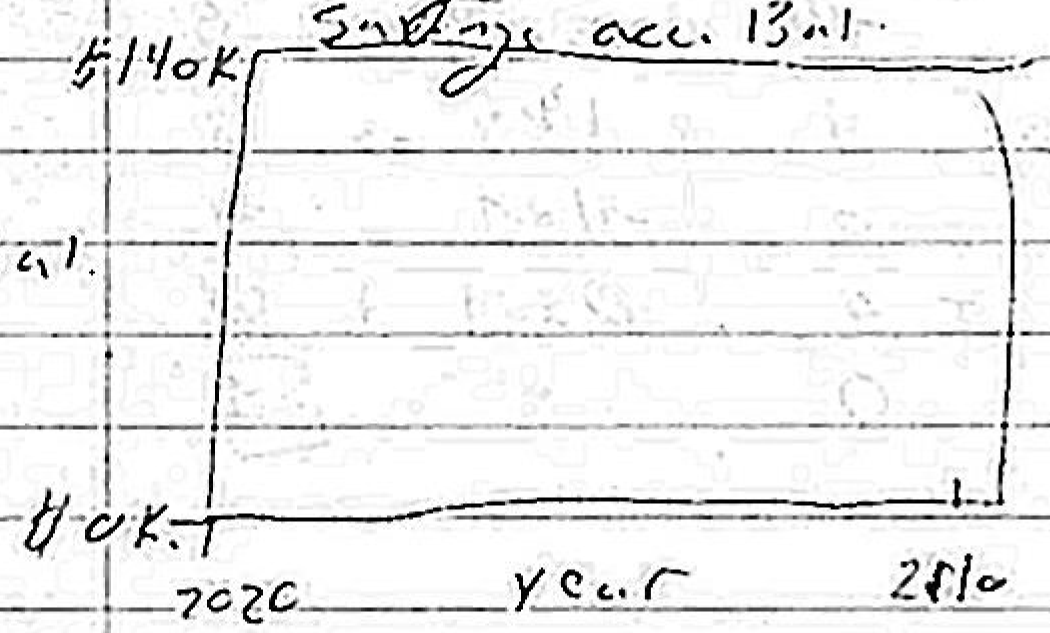
```
>> x*y
```

$$\begin{array}{c} x \\ \begin{bmatrix} -1 \\ 2 \\ 3 \end{bmatrix} \end{array} \text{①-Row} \times \begin{array}{c} y \\ \begin{bmatrix} 5 & 2 & 3 \end{bmatrix} \end{array} \text{① col.}$$

$$\overset{\text{yes}}{1 = 1} = \begin{bmatrix} -1 * 5 \\ 2 * 2 \\ 3 * 3 \end{bmatrix} \Rightarrow \begin{matrix} -5 \\ 4 \\ 9 \end{matrix} \Rightarrow \begin{matrix} -5 \\ 4 \\ +9 \\ \hline 8 \end{matrix}$$

Program Control

Writing a For Loop.



New Balance = (1 + r)(Current Balance)

MATLAB Comm.

```
r = 0.02
balance = zeros(1,100);
balance(1) = 20,000
balance(2) = (1+r)*balance(1);
balance(3) = (1+r)*balance(2);
                 :
balance(5) = (1+r)*balance(4);
                 % Automate w/ 4-Loop.
for K = 1:99
        balance(K+1) = (1+r)*balance(K);
end
```

MATLab (Program Control) 10/14

Functions: File

MATLAB Editor

```
ratio = 1/2;
 N = 5;
 aSum = geoSum(ratio, N)
 function s = geoSum(r, n)
  % geoSum.m sums the first n terms of a
 % geometric series w/common ratio r.
 if r == 1
   s = n;
else
  s = (1-r^n)/(1-r);
end.
```

MATLAB Comm.

```
>> aSum = geoSum(2,4)
Undefined funct. or
Variable 'geoSum'.
```

geoSum.m

```
function s = geoSum(r, n)
  % geoSum.m sums the first n terms of a
  % geometric series with common ratio r
if r == 1
   s = n;
else
  s = (1-r^n)/(1-r);
end
end.
```

Current Folder

- chebyNode.m
- fibonaccis.m
- geoSum.m
- legendrePoly.m

Now when we Pass value in comm.

MATLAB Commands

```
>> aSum = geoSum(2,4)
   aNewSum = 15
```

T aNewSum 15

Other Topics

Calculating Eigenvalues and Eigenvectors

eigenvector

$Av = \lambda v$

eigenvalue

can have many eigenvalue pairs

$Av = \lambda v$

λ_1, v_1

λ_2, v_2

$\vdots$

MATLAB Function

eig(A)

ex. MATLAB Comm.

```
>> [V, D] = eig(A);
```

⊞ A

$$\begin{bmatrix} 3 & -2 & 0 \\ 1 & 0 & 0 \\ -1 & 1 & 3 \end{bmatrix}$$

⊞ V

$$\begin{bmatrix} 0 & -0.7071 & \boxed{\begin{matrix} -0.8165 \\ -0.4082 \\ -0.4082 \end{matrix}} \\ 0 & -0.7071 & \\ 1.0000 & 0 & \end{bmatrix}$$

⊞ D

$$\begin{bmatrix} 2 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & \boxed{2} \end{bmatrix}$$

```
>> v2 = [1; 1; 0];
>> A * v2 - 1 * v2;
ans = 0
      0
      0
```

$Av_2 = \lambda_2 v_2$

$v_2 \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$ $\lambda_2 = 1$

$v_3 = ?$

Structure Arrays

[E] structureVariable
- field1 [abc] 1x10 char
- field2 [▦] 100x1 double
- field3 [✓] 10x10 logical
- ⋮

MATLAB Comm.

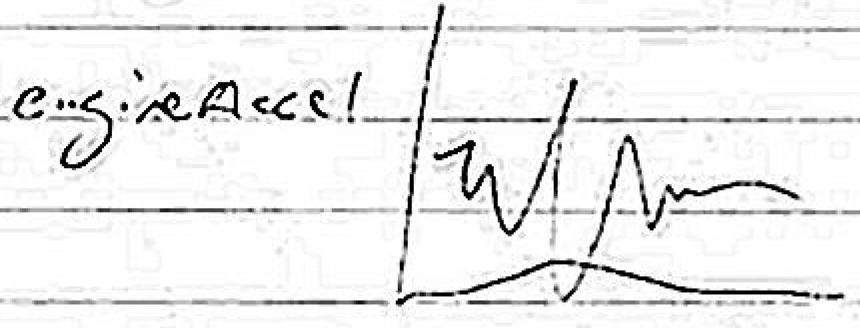
```
>> crashTest.Make = 'Honda Fit';
>> crashTest.Speed = 35;
>> crashTest.Time = Time;
>> crashTest.NeckForce = neckSensor;
>> crashTest.Engine = readEng;
```

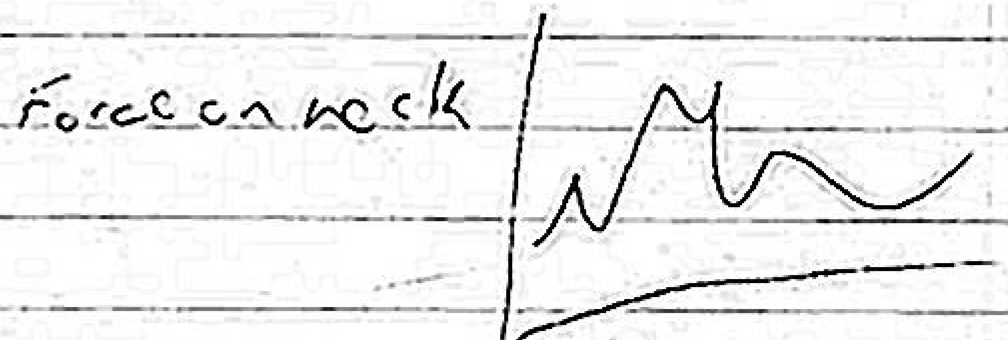
[E] crashtest 1x1 struct
- Make: 'Honda Fit'
- Speed: 35
- Time [3000 x 1 double]
- NeckF [3000 x 1 double]

```
% convert speed from mph to km/h
>>crashTest.Speed = mph2kmph(crashTest.speed)
crashTest.Speed = 58.3
```

[E] crashTest =

```
% input all data w/out seperate data.
>> analyzeCollision(crashTest)
```





①

normally — Cols x Rows

10/20

Identity Matrix

$1 \cdot x = x$

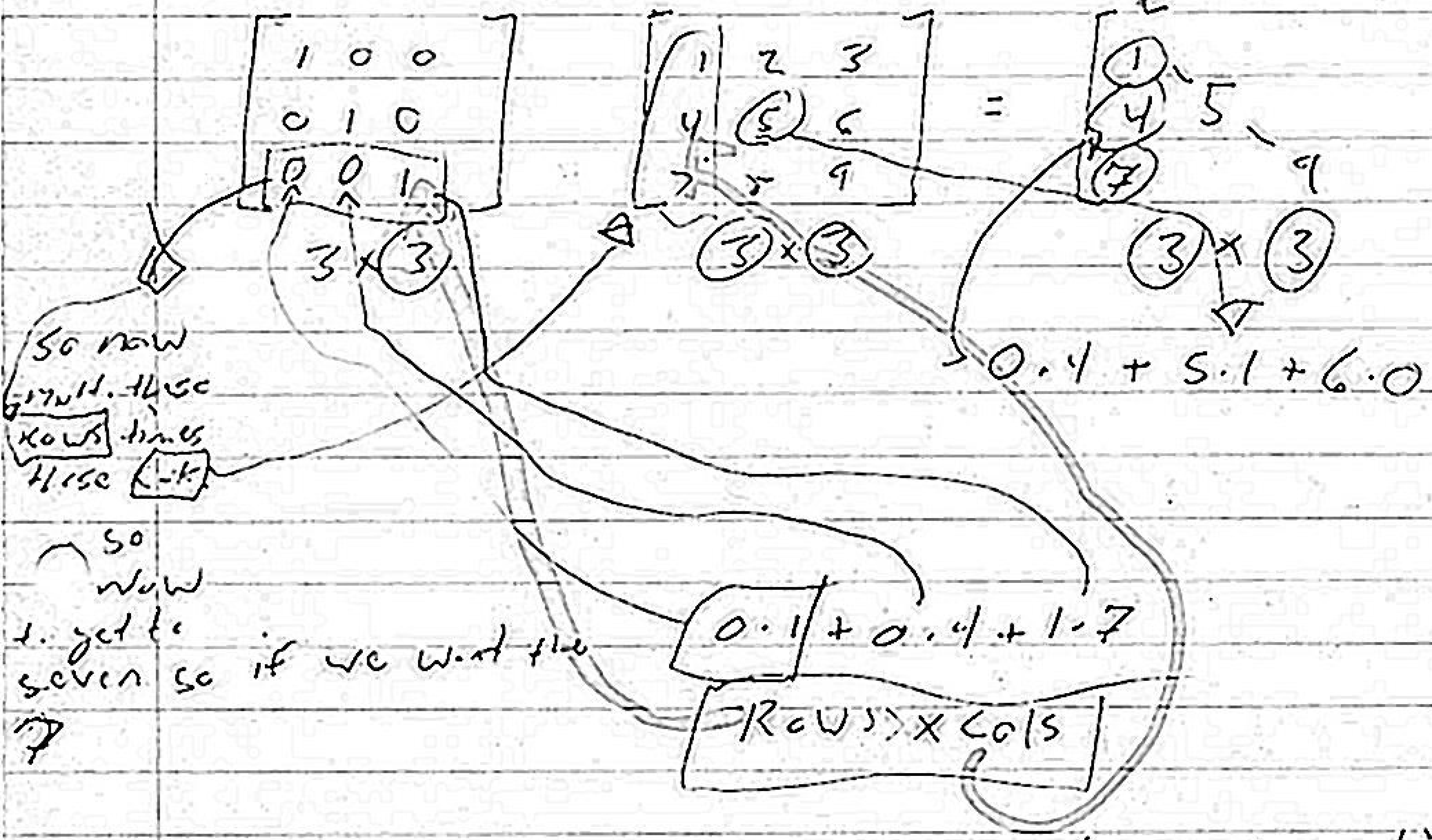
identity

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$$

$IA = A$

$(I)\cdot(A)$

$1\cdot1 + 0\cdot4 + 0\cdot7$



$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix} = \begin{bmatrix} 1 & & \\ 4 & 5 & \\ 7 & & 9 \end{bmatrix}$$

$3 \times 3 \qquad 3 \times 3 \qquad 3 \times 3$

$0\cdot4 + 5\cdot1 + 6\cdot0$

so now mult. these Rows times these Cols

so now to get to seven so if we want the 7

$0\cdot1 + 0\cdot4 + 1\cdot7$

Rows x Cols

You simply have ones going down the diagonal starting at the top left to right

$$I = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Inverse of 3 x 3 matrix: (A^{-1}) 10/20

$$\begin{matrix} & & 15 & 30 & 0 \end{matrix}$$

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 5 \\ 5 & 6 & 0 \end{bmatrix} \begin{matrix} 1 & 2 \\ 0 & 1 \\ 5 & 6 \end{matrix}$$

$\uparrow$ $\quad$ 0 50 0

$\uparrow$

- creates 3 diagonals
- mult. numbers in all 3 diagonals

$(1 \cdot 1 \cdot 0) = 0$
$(2 \cdot 5 \cdot 5) = 50$
$(3 \cdot 0 \cdot 6) = 0$
$\Big\}$ red numbers

- Next mult. diagonals going opposite way

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 5 \\ 5 & 6 & 0 \end{bmatrix} \begin{matrix} 1 & 2 \\ 0 & 1 \\ 5 & 6 \end{matrix}$$

$(5 \cdot 1 \cdot 3) = 15$
$(6 \cdot 5 \cdot 1) = 30$
$(0 \cdot 0 \cdot 2) = 0$
$\Big\}$ green numbers

- Now take the red numbers and subtract from green numbers

$|A| = (0 + 50 + 0) - (15 + 30 + 0)$ // Finding Determinate

$50 - 45$

$\boxed{5}$

$|A| = \boxed{5}$

- Now create a new matrix called N.

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 5 \\ 5 & 6 & 0 \end{bmatrix} \qquad N = \begin{bmatrix} \begin{vmatrix} 1 & 5 \\ 6 & 0 \end{vmatrix} & \begin{vmatrix} 0 & 5 \\ 5 & 0 \end{vmatrix} & \begin{vmatrix} 0 & 1 \\ 5 & 6 \end{vmatrix} \\ \begin{vmatrix} 2 & 3 \\ 6 & 0 \end{vmatrix} & \begin{vmatrix} 1 & 3 \\ 5 & 0 \end{vmatrix} & \begin{vmatrix} 1 & 2 \\ 5 & 6 \end{vmatrix} \\ \begin{vmatrix} 2 & 3 \\ 1 & 5 \end{vmatrix} & \begin{vmatrix} 1 & 3 \\ 0 & 5 \end{vmatrix} & \begin{vmatrix} 1 & 2 \\ 0 & 1 \end{vmatrix} \end{bmatrix}$$

cross out Row and column

mult this way

$$N = \begin{bmatrix} \begin{vmatrix} 1 & 5 \\ 6 & 0 \end{vmatrix} & \begin{vmatrix} 0 & 5 \\ 5 & 0 \end{vmatrix} & \begin{vmatrix} 0 & 1 \\ 5 & 6 \end{vmatrix} \\ \begin{vmatrix} 2 & 3 \\ 6 & 0 \end{vmatrix} & \begin{vmatrix} 1 & 3 \\ 5 & 0 \end{vmatrix} & \begin{vmatrix} 1 & 2 \\ 5 & 6 \end{vmatrix} \\ \begin{vmatrix} 2 & 3 \\ 1 & 5 \end{vmatrix} & \begin{vmatrix} 1 & 3 \\ 0 & 5 \end{vmatrix} & \begin{vmatrix} 1 & 2 \\ 0 & 1 \end{vmatrix} \end{bmatrix}$$

these are the determintes

$$(1 \times 0) - (5 \cdot 6) = -30$$

$$(0 \times 0) - (5 \cdot 5) = -25$$

$$N = \begin{bmatrix} -30 & -25 & -5 \\ -18 & -15 & -4 \\ 7 & 5 & 1 \end{bmatrix}$$

- Next step alternate positive and negative signs

of Alt. sign.

Now start w/negative

$$N = \begin{bmatrix} + -30 & - -25 & + -5 \\ - -18 & + -15 & - -4 \\ + 7 & -5 & + 1 \end{bmatrix} \Rightarrow N = \begin{bmatrix} -30 & 25 & -5 \\ 18 & -15 & 4 \\ 7 & -5 & 1 \end{bmatrix}$$

(switch places)

(red diagonal stays the same)

$$N = \begin{bmatrix} -30 & 18 & 7 \\ 25 & -15 & -5 \\ -5 & 4 & 1 \end{bmatrix} \Rightarrow A^{-1} = \frac{1}{|A|}[N]$$

$$A^{-1} = \frac{1}{5}\begin{bmatrix} -30 & 18 & 7 \\ 25 & -15 & -5 \\ -5 & 4 & 1 \end{bmatrix} \Rightarrow A^{-1} = \begin{bmatrix} -6 & 18/5 & 7/5 \\ 5 & -3 & -1 \\ -1 & 4/5 & 1/5 \end{bmatrix}$$

Ex: Solve the Matrix $AX = B$ (3x3)

Find Inverse $\qquad X = A \backslash B$ $\qquad$ 10/21

$f(x) = 3x - 2$

$y = 3x - 2$ solve for X

$y + 2 = 3x \Rightarrow \frac{y+2}{3} = x \Rightarrow x = \frac{y}{3} + \frac{2}{3}$

① so $f^{-1}(x) = \frac{x}{3} + \frac{2}{3}$

Back to Matrix Equation: $AX = B$

Solve the matrix $AX = B$ for X

$$A = \begin{bmatrix} -1 & -11 & -3 \\ 1 & 1 & 0 \\ 2 & 5 & 1 \end{bmatrix}, \quad B = \begin{bmatrix} -37 \\ -1 \\ 10 \end{bmatrix} \quad \text{so} \quad \begin{aligned} AX &= B \\ A^{-1} \cdot AX &= A^{-1} \cdot B \\ IX &= A^{-1} \cdot B \\ X &= A^{-1} \cdot B, \end{aligned}$$

1st ① write a augmented matrix $\qquad$ identity matrix

$$\begin{bmatrix} -1 & -11 & -3 \\ 1 & 1 & 0 \\ 2 & 5 & 1 \end{bmatrix} X = \begin{bmatrix} -37 \\ -1 \\ 10 \end{bmatrix}$$

$$[A\,I] \rightarrow [I\,A^{-1}]$$

identity matrix

$$\left[\begin{array}{ccc|ccc} -1 & -11 & -3 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 & 1 & 0 \\ 2 & 5 & 1 & 0 & 0 & 1 \end{array}\right] \rightarrow [I\,A^{-1}]$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & -4 & 3 \\ 0 & 1 & 0 & -1 & 5 & -3 \\ 0 & 0 & 1 & 3 & -17 & 10 \end{array}\right]$$

A^{-1} - Inverse of A

stands for identity matrix

*Key

$AX = B$

$A^{-1} \cdot AX = A^{-1} \cdot B$

$\textcircled{I} X = A^{-1} B$

$X = A^{-1} \cdot B$

$$// \; A^{-1} \cdot AX = A^{-1} \cdot B$$

$$\underbrace{\begin{bmatrix} 1 & -4 & 3 \\ -1 & 5 & -3 \\ 3 & -17 & 10 \end{bmatrix}}_{A^{-1}} \cdot \underbrace{\begin{bmatrix} -1 & -11 & -3 \\ 1 & 1 & 0 \\ 2 & 5 & 1 \end{bmatrix}}_{A} X = \begin{bmatrix} 1 & -4 & 3 \\ -1 & 5 & -3 \\ 3 & -17 & 10 \end{bmatrix} \begin{bmatrix} -37 \\ -1 \\ 10 \end{bmatrix}$$

3 × 3 = 3 × 1

cols = Rows

Result 3 × 1 matrix

$$Ix = \begin{bmatrix} 1 & -4 & 3 \\ -1 & 5 & -3 \\ 3 & -17 & 10 \end{bmatrix} \begin{bmatrix} -37 \\ -1 \\ 10 \end{bmatrix}$$

$$Ix = \begin{bmatrix} (1 \times -37) + (-4 \times -1) + (3 \times 10) \\ (-1 \times -37) + (5 \times -1) + (-3 \times 10) \\ (3 \times -37) + (-17 \times -1) + (10 \times 10) \end{bmatrix} \Rightarrow \begin{bmatrix} -37 + 4 + 30 \\ 37 + (-5) + (-30) \\ (-111) + 17 + 100 \end{bmatrix}$$

$$IX = \begin{bmatrix} -3 \\ 2 \\ 6 \end{bmatrix} \rightarrow X = \begin{bmatrix} -3 \\ 2 \\ 6 \end{bmatrix}$$

Ex: Solve the Matrix Equation AX=B (3x3):

https://www.youtube.com/watch?v=t4e6R-bDOGU

Inverse of 3x3 matrix:

https://www.youtube.com/watch?v=pKZyszzmyeQ

Identity matrix | Matrices | Precalculus | Khan Academy:

https://www.youtube.com/watch?time_continue=1&v=3cnIa0fYJkY

Finding the Inverse of an n x n Matrix Using Row Operations (Did not take notes):

https://www.youtube.com/watch?v=HwRRdG_E4Yo

Solve Linear Equations with MATLAB:

https://www.youtube.com/watch?v=-m6qtOrNPkQ&t=267s