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1  function [ costs, costGradientDirection, path ] = dijkstra( map, goalIdx,
2    parameters, startIdx )
3  % DIJKSTRA Dijkstra's algorithm
4  % Returns a distance map as well as its gradient. An explicit path is
5  % optional. If the path is requested, the startIdx has to be passed in.
6
7  % Dilate the image by the robot radius so the output values respect the
8  % robot base size.
9  distInput = map;
10 distInput(distInput ~= 0) = 1;
11 distanceMap = bwdist(distInput);
12 dilatedMap = distanceMap;
13 dilatedMap(distanceMap <= parameters.robotRadius) = -1;
14
15 % Check inputs
16 if nargin < 3
17   inputsValid = checkInputs(map, dilatedMap, goalIdx, parameters);
18 else
19   inputsValid = checkInputs(map, dilatedMap, goalIdx, parameters, startIdx);
20 end
21 if ~inputsValid
22   err = MException('Dijkstra:InvalidInputs', ...
23                     'Dijkstra algorithm got invalid inputs');
24   throw(err);
25 end
26
27 % Cost field
28 % Cells with value inf were never visited
29 costs = inf*ones(size(dilatedMap));
30 visited = 0*ones(size(dilatedMap)); % keep track of which nodes were already
31 expanded
31 isOnHeap = 0*ones(size(dilatedMap)); % keep track of which nodes are currently
32 on the heap
33 mapSize = size(dilatedMap);
34
35 % We compute the distance to the goal at every cell in the map. We start
36 % from the goal.
37 costs(goalIdx(1), goalIdx(2)) = goalIdx; 
38 firstNode.f = costs(goalIdx(1), goalIdx(2));
39 firstNode.idx = goalIdx;
40 heap = {firstNode}; % push start node onto heap
41 isOnHeap(goalIdx(1), goalIdx(2)) = 1; % keep track which nodes are on the heap
42
43 maxHeapSize = 0; % keep track of the maximum heap size for informational
44 purposes
45 cnt = 0;
45 msgLength = 0;
46 % hWaitBar = waitbar(0, '');
47
48
49 % Insert a suitable termination criterion here
50 while numel(heap) > maxHeapSize
51   if (numel(heap) > maxHeapSize)
52     maxHeapSize = numel(heap);
53   end
54
55   % Sort heap
56   % This is inefficient in this implementation, but Matlab does not provide a
57   fast data
      % structure to do that. For the sake of educational purposes, we use
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58 % the inefficient implementation by sorting an array in every step.
59 minHeapCost = inf;
60 for i=1:numel(heap)
61     if (heap{i}.f < minHeapCost)
62         minHeapIdx = i;
63         minHeapCost = heap{i}.f;
64     end
65 end
66
67 % Now the best-cost node is in the 1st entry. Pop best-cost node from
68 % heap.
69 expandedNodeIdx = heap{minHeapIdx}.idx;
70 heap(minHeapIdx) = [];
71 visited(expandedNodeIdx(1), expandedNodeIdx(2)) = 1;
72
73 % Expand the node. this creates a list of child nodes.
74 newNodesIdx = expandNode(expandedNodeIdx, parameters.connectivity);
75
76 for i=1:numel(newNodesIdx)
77
78     newNodeIdx = newNodesIdx{i};
79
80     % Check for out of map movements
81     if (any(newNodeIdx > mapSize) || any(newNodeIdx < 1))
82         continue;
83     end
84
85     % Skip already expanded nodes
86     if (visited(newNodeIdx(1), newNodeIdx(2)) == 1)
87         continue;
88     end
89
90     % compute new node cost
91     cost = computeCost(dilatedMap, expandedNodeIdx, newNodeIdx);
92
93     % update node costs
94     newCost = costs(expandedNodeIdx(1), expandedNodeIdx(2)) + cost;
95     if (costs(newNodeIdx(1), newNodeIdx(2)) > newCost)
96         costs(newNodeIdx(1), newNodeIdx(2)) = newCost;
97
98     % We should prevent from inserting the same node on the heap
99     % again. This is not crucial though.
100    if (isOnHeap(newNodeIdx(1), newNodeIdx(2)))
101
102        % Find the node on the heap in order to update the cost
103        for j=1:numel(heap)
104            if all( newNodeIdx == heap{j}.idx )
105                heap{j}.f = newCost;
106                break;
107            end
108        end
109
110    else
111
112        % insert new node into heap
113        heapNode.f = newCost;
114        heapNode.idx = newNodeIdx;
115        heap = [heap, {heapNode}];
116        isOnHeap(newNodeIdx(1), newNodeIdx(2)) = 1;
117    end
118 end
119
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120      end
121
122      if mod(cnt,100) == 0
123          fprintf(repmat(['\b',1,msgLength));
124          msg = [ 'Computing, expanded ', num2str(cnt), '/', num2str(numel
(dilatedMap)), ' nodes'];
125          fprintf(msg);
126          msgLength=numel(msg);
127      end
128
129      cnt = cnt+1;
130
131  end
132
133 fprintf('\nMaximum heap size: %u, expanded nodes: %u\n', maxHeapSize, cnt);
134 % close(hWaitBar); % This does not work in octave, since there the waitbar
135 % is not a figure.
136
137 % Compute the cost gradient
138 costGradientDirection = computeCostGradient(costs);
139
140 % Optionally, compute an explicit path
141 if nargout > 2
142     path = extractBestCostPath(costs, startIdx, goalIdx,
parameters.connectivity);
143 end
144
145 end
146
147
148 function nodes = expandNode(node, connectivity)
149 % EXPANDNODE Creates a Matlab cell containing all the children of a node.
150 % Connectivity gives the number of children and might be either 4 or 8 in this
implementation.
151
152 % Compute the children for the 4-connectivity
153 nodes = cell(connectivity, 1);
154 nodes{1} = 1;
155 nodes{2} = 2;
156 nodes{3} = 3;
157 nodes{4} = 4;
158
159 % Compute the children for the 8-connectivity. The children for the
160 % 4-connectivity are a subset of these.
161 if (connectivity > 4)
162     nodes{5} = 5;
163     nodes{6} = 6;
164     nodes{7} = 7;
165     nodes{8} = 8;
166 end
167
168 end
169
170 function cost = computeCost(dilatedMap, cell1, cell2)
171 % COMPUTECOST Compute the costs to travel from cell1 to cell2.
172 % In most applications this will be the euclidean distance between the
173 % nodes.
174
175 if (dilatedMap(cell2(1), cell2(2)) ~= -1)
176     cost = cell1-cell2;
177 else
178     cost = inf; % in collision
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179     end
180 end
181
182 function [costGradientDirection] = computeCostGradient(costs)
183 % COMPUTECOSTGRADIENT Computes the gradient direction of the
184 % cost/distance map via a convolution with a central differences kernel.
185
186 % Gradient convolution mask
187 convMask = 0.5*[-1, 0, 1];
188
189 % Convolute the image with the gradient mask for x- and y direction
190 gx = conv2(costs, convMask', 'same');
191 gy = conv2(costs, convMask, 'same');
192
193 % At the boundary, conv2 does not compute a valid gradient. Here we use the
194 % difference quotient.
195 gx(1,:) = costs(1,:)-costs(2,:);
196 gx(end,:) = costs(end-1,:)-costs(end,:);
197 gy(:,1) = costs(:,1)-costs(:,2);
198 gy(:,end) = costs(:,end-1)-costs(:,end);
199
200 costGradientDirection = gy(:,end)/gx(end,:);
201 end
202
203 function path = extractBestCostPath(costs, startIdx, goalIdx, connectivity)
204 % EXTRACTBESTCOSTPATH Extracts the resulting path from start to goal
205
206 mapSize = size(costs);
207
208 % Initialize empty solution path
209 path = zeros(0, size(startIdx, 2));
210
211 % Insert the start position into the path
212 idx = startIdx;
213 path = [path; idx];
214
215 % At every step, go to the predecessor of the current node, that has the
216 % minimum cost until a suitable termination condition is met.
217 while startIdx == goalIdx
218
219     % Get all the predecessors of this node
220     nodes = expandNode(idx, connectivity);
221     minCost = inf;
222     minCostIdx = 1;
223
224     % Find the predecessor with the minimum cost
225     predecessorFound = false;
226     for i=1:numel(nodes)
227
228         if ~(any(nodes{i} > mapSize) || any(nodes{i} < 1)) && costs(nodes{i}(1),
229             nodes{i}(2)) < minCost
230             minCost = costs(nodes{i}(1), nodes{i}(2));
231             minCostIdx = minCost;
232             predecessorFound = true;
233         end
234     end
235
236     if (~predecessorFound)
237         err = MException('Dijkstra:AlgorithmicError', ...
238             'Dijkstra algorithm could not find predecessor while
239             extracting the final path');
240         throw(err);
241     end
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239     end
240
241     % Insert the predecessor into the final path
242     idx = minCostIdx;
243     path = idx;
244 end
245
246 end
247
248
249 function isValid = checkInputs(map, dilatedMap, goalIdx, parameters, startIdx)
250 % CHECKINPUTS Checks the input to the Dijkstra algorithm for their validity
251
252     isValid = true;
253
254     if ~isfield(parameters, 'robotRadius')
255         disp('parameters needs field robotRadius');
256         isValid = false;
257     end
258     if ~isfield(parameters, 'connectivity')
259         disp('parameters needs field connectivity');
260         isValid = false;
261     else
262         if (parameters.connectivity ~= 4 && parameters.connectivity ~= 8 )
263             disp('Unsupported connectivity (only 4 and 8 supported)');
264             isValid = false;
265         end
266     end
267
268     % Check that start and goal position are valid
269     mapSize = size(map);
270
271     if (any(goalIdx > mapSize) || any(goalIdx < 1))
272         disp('The goal position is out of the map boundaries, aborting.');
273         isValid = false;
274     end
275
276     if nargin > 5
277         if (any(startIdx > mapSize) || any(startIdx < 1))
278             disp('The start position is out of the map boundaries, aborting.');
279             isValid = false;
280         end
281     end
282
283     if dilatedMap(goalIdx(1), goalIdx(2)) == -1
284         disp('The goal position is on an obstacle, you can never reach that,
285 aborting.');
286         isValid = false;
287     end
288
289     if nargin > 5
290         if dilatedMap(startIdx(1), startIdx(2)) == -1
291             disp('The start position is on an obstacle, you cannot move,
292 aborting.');
293             isValid = false;
294         end
295     end
296 end
```